Computer Science 162 Final Practice – April, 2018 Name:__________________

Definition: $f(n) \in O(g(n))$ iff $\exists$ positive c, n_0 such that $f(n) \leq c \cdot g(n) \quad \forall n \geq n_0$.

1. Prove that $3n^6 + 4n^4 \in O(n^6)$, using the definition of Big Oh.
2. Prove that $2n^5 + n^2 + 10 \in O(n^5)$, using the definition of Big Oh.
3. How many times does `sum++` get executed in the following code fragments? Show your work, and report the big Oh running time of the code. You may assume that n is a power of 2.

```
for (i=1; i<=n; i++)
    for (j=i; j<=n; j++)
            sum++;
```

```
for (i=1; i<=n; i++)
    for (j=i; j<=n; j++)
        for (k=1; k<= n; k = k*2)
            sum++;
```

4. Prove that $3n^3 + n \log n + 1 \in O(n^3)$.
5. Convert the binary number 1100010 into decimal.
6. Convert the decimal number 2919 into hexidecimal (base 16).
7. Perform 2's complement addition on the two seven-bit binary numbers given below, yeilding an answer in binary form as the 2's complement representation of the result.
   1011111
   0111110
8. Perform 2's complement addition on the following and leave your answer in binary form as the 2's complement representation of the result.
   1001011
   0111011
9. In eight-bit 2's complement, what is the number 11010101? Give your answer in decimal (base 10).

10. The MARIE instructions are the following:

Bin	Hex	Instruction	Description
0001	1	Load X	Load contents of address X into AC
0010	2	Store X	Store the contents of AC at address X
0011	3	Add X	Add the contents of Memory[X] to contents of the AC
0100	4	Subtract X	Subtract the contents of Memory[X] from contents of AC
0101	5	Input	Enter the value from keyboard into AC
0110	6	Output	Output the value in AC to the display
0111	7	Halt	Terminate the program
1000	8	Skipcond	Skip the next instruction on condition
1001	9	Jump X	Load the value X into PC
0000	0	JnS X	Jump and Store: Stores value of PC at Address X then loads value X+1 into PC
1011	C	JumpI X	Uses the value at X as address to jump to

Skipcond checks the value of the low 12 bits of the IR (instruction register):
if they are hex 800, control skips if the AC > 0
if they are hex 400 control skips if AC $= 0$, and
if they are hex 000 control skips if AC < 0.

(a) Write a subroutine TwoXMinusY that takes values stored at location X and Y, and computes the value of 2*X-Y, and places it in TwoXMinusYResult.

(b) Write a main program that repeatedly gets two numbers from the user, loads them into X and Y, and runs TwoXminusY, and outputs the result. The program should do this repeatedly until the first of the two numbers is zero, at which point it halts.

11. Recall `append` is a prolog predicate such that `append(L1, L2, Result)` is satisfied if Result is the concatenation of L1 with L2 (L1 appearing first).

```
append([],L,L).
append([H|T],L,[H|Tresult]):- append(T,L,Tresult).
```

Consider `subseq(L1,L2)`, which is satisfied if L2 appears as a contiguous subsequence of L1 (so subseq([a,b,b,b,a],[b,b,b]) is satisfied, but subseq([a,b,a,b,a,b,a],[b,b,b]) is not).

Write the code for the predicate subseq.

12. Consider non-contiguous subsequence predicate, that is, L2 is a non-contiguous subsequence of L1 if all the elements of L2 appear in L1 in the same order. Eg, ncSubseq([a,b,c,d,a,b],[a,b,d,b]) is true. Write ncSubseq(L1,L2).

13. Write a Prolog predicate called interleave(L1,L2,L), where list L is the "interleave" of lists L1 and L2: that is L has as its first element, the first element of L1, followed by the first element of L2, followed by the second element of L1, followed by the second element of L2, and so on. When one list runs out of elements, then the remaining list provides the remainder of the elements.

For example,

$L1 = [a1, a2, a3]$ and

$L2 = [b1, b2, b3, b4, b5, b6]$, the list L will have the value

$[a1, b1, a2, b2, a3, b3, b4, b5, b6]$.

14. Given

$$F(x, y, z) = xy + \overline{xy} + \overline{y}z$$

Draw the combinational circuit that **directly** implements the Boolean expression F.

15. For the function described by the following table, group the 1's together to form a good Karnaugh map, by drawing "boxes" of 1's on the table. Then write the associated Boolean algebra expression. To receive full marks, your expression should minimize the number of literals (i.e., negated or unnegated variable instances) in the sum-of-products formula that results.

16. Suppose you have 64 GiB of memory that is word-addressable, and the words are 32 bits long. How many bits long should the addresses be?

17. Describe briefly what is meant by the Fetch, Decode and Execute cycle. Include an indication of when the system will deviate from the cycle.

18. Fill in the blanks. In a Von Neumann architecture, both ____________ and ____________ can be stored in memory. To give each process the illusion that it has a CPU all to itself, the operating system employs a scheme called ____________. As a part of the CPU, the ____________ carries out logic operations and arithmetic operations. A ________________________ is a hardware device that stores binary data; several are located on the processor itself; the size of it, measured in bits, is called the ____________ -size, and is a characteristic of the architecture. On the CPU, the ____________ is the "traffic manager" and tells the ALU what to do by turning on the correct circuitry. The ____________ is the set of wires that acts a a data path to connect system components.

19. In unix, what is a 'pipe'? How is it invoked, and what is its function?

20. Give Regular Expressions for the languages accepted by the following DFA's.

21. Draw a Finite Automaton for the language of all strings over $\{a, b\}$ that do not contain the string *bbab*.

22. Give regular expressions for the following languages:

    (a) $\{w \in \{a, b\}^* :$ every block of *a*s is of even length $\}$

    (b) $\{w \in \{a, b\}^* : w$ contains the substring *aab* and the substring *bba* $\}$

    (c) $\{w \in \{a, b\}^* : w$ does not contain the substring *ba* nor the substring *ab* $\}$

23. Describe in Turing-Machine pseudocode a TM that accepts $\{0^a1^b : a > b\}$.