VANCOUVER ISLAND UNIVERSITY
CSCI 260 — MIDTERM EXAMINATION
15 October 2019, 11:30 — 12:50

Duration: 80 Minutes **Instructors:** H. Liu

TO BE ANSWERED IN BOOKLETS

Instructions

- Students must count the number of pages in this examination paper before beginning to write, and report any discrepancy immediately to the invigilator.
- This examination paper consists of 4 pages.
- This is a CLOSED BOOK examination. You are allowed to bring one piece of letter-sized and double-sided notes.
- Calculators, electronic devices and network connection access are NOT permitted.
- Remember to state any assumptions and show rough work.
- Note carefully the weight of each question, and answer appropriately.
- Attempt all questions. All questions relate to material covered in the lectures, labs and assignments.

Reference

The Master Theorem: Let $f(n)$ and $T(n)$ be defined as:

$$T(n) = \begin{cases} c & \text{if } n < d \\ aT(n/b) + f(n) & \text{if } n \geq d. \end{cases}$$

where $d \leq 1$ is an integer constant, $a > 0, c > 0$, and $b > 1$ are real constants and $f(n)$ is a function that is positive for $n \geq d$.

1. If there is a small constant $\varepsilon > 0$, such that $f(n)$ is $O(n^{log_b a - \varepsilon})$, then $T(n)$ is $\Theta(n^{log_b a})$.

2. If there is a constant $k \geq 0$, such that $f(n)$ is $\Theta(n^{log_b a} log^k n)$, then $T(n)$ is $\Theta(n^{log_b a} log^{k+1} n)$.

3. If there are small constants $\varepsilon > 0$ and $\delta < 1$, such that $f(n)$ is $\Omega(n^{log_b a + \varepsilon})$ and $af(n/b) \leq \delta f(n)$, then $T(n)$ is $\Theta(f(n))$.

1. (10 Marks)

   (a) List the main operations of the ADT Dictionary. For each operation, describe its required parameters and its functionality.

   (b) Suppose that a sorted array is used to implement the dictionary, write a C++ function to illustrate the algorithm that implements the operation of `look up` an object with a given key (of integer type).

2. (10 Marks)

   (a) What is a heap? (Hint: describe the two main properties of a heap.)

   (b) Heap can be used to implement the ADT priority queue. Compared with using sorted array, what is the advantage and disadvantage of using heap as the ADT priority queue's implementation?

   (c) In which heap operation do we need to perform the `upHeap` operation? What is the purpose of performing the `upHeap` operation?

   (d) Describe the algorithm of the `upHeap` operation.

3. (9 Marks)

   (a) Describe a typical application that uses a hash table as its implementation.

   (b) What is a collision in hash table? Why is collision a problem in hash table?

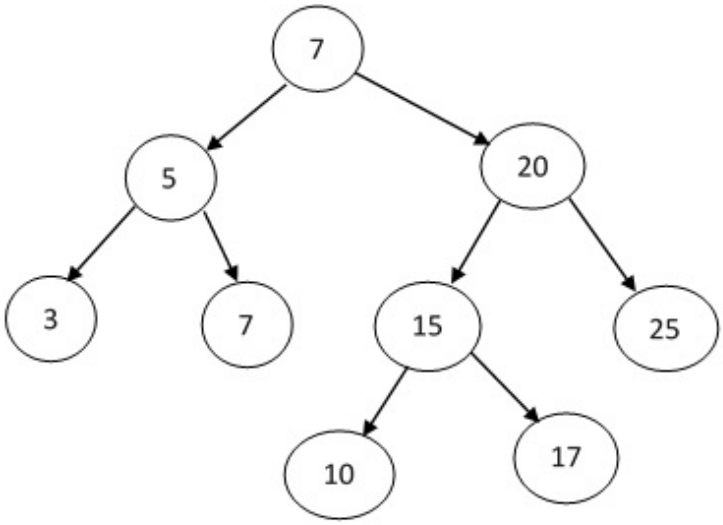
   (c) Describe the two distinct methods commonly used to solve the collision problem in hash table.

4. (5 Marks) Given the following C++ function-like algorithm:

```
int fun(int n)
{
    if (n <= 1)
        return 1;
    return n*fun(n-1);
}
```

   Write the recurrence equation for the running time of the above algorithm.

5. (6 Marks) An AVL tree that stores integer numbers in its nodes is shown below.



Inserting numbers in which range (or ranges) would cause a double rotation? Show the tree (or trees) after the rotation (You can use x to represent the newly inserted number).

6. (10 Marks) Given the following recurrence equation that defines T(N) as:

$$T(N) = \begin{cases} 2 & \text{if } N = 1 \\ T(N-1) + 2N & \text{if } N > 1. \end{cases}$$

Prove, by induction, that $T(N) = N^2 + N$.

====== END OF EXAM QUESTIONS ======