

Computer Science CSCI 355

Digital Logic and Computer Organization

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Minimal SOP/POS

○ DeMorgan's Theorem

- $\overline{AB} = \overline{A} + \overline{B}$
- $\overline{A + B} = \overline{A}.\overline{B}$

minimal POS for $f = \overline{\text{minimal SOP for } \overline{f}}$

minimal SOP for $f = \overline{\text{minimal POS for } \overline{f}}$

Example

f		c d			
		00	01	11	10
a b	00	0	0	0	1
	01	1	1	0	1
	11	1	1	0	1
	10	0	1	0	0

$$f = bc' + bd' + ac'd + a'cd'$$

Example cont.

		c d			
		00	01	11	10
f	a b	0	0	0	1
	01	1	1	0	1
	11	1	1	0	1
	10	0	1	0	0

$$f' = cd + ab'd' + a'b'c'$$

Example cont.

$$\bar{f} = cd + \bar{a} \bar{b} \bar{c} + a \bar{b} \bar{d}$$

$$\Rightarrow \overline{\bar{f}} = \overline{cd + \bar{a} \bar{b} \bar{c} + a \bar{b} \bar{d}}$$

$$\Rightarrow f = (\overline{cd})(\overline{\bar{a} \bar{b} \bar{c}})(\overline{a \bar{b} \bar{d}})$$

$$\Rightarrow f = (\bar{c} + \bar{d})(a + b + c)(\bar{a} + b + d)$$

Example cont.

a	b	c	d	m	M
0	0	0	0	$a'b'c'd'$	$a+b+c+d$
0	0	0	1	$a'b'c'd$	$a+b+c+d'$
0	0	1	0	$a'b'cd'$	$a+b+c'+d$



Example cont.

		c d			
		00	01	11	10
f	a b	0	0	0	1
	01	1	1	0	1
	11	1	1	0	1
	10	0	1	0	0

$$f = (c'+d') (a+b+c) (a'+b+d)$$

Don't Cares

- e.g. $f = \sum m(1, 4, 5) + \sum md(0, 7)$

		bc			
		00	01	11	10
a	0	-	1	0	0
	1	1	1	-	0

$$f = b'$$

Multiple Outputs

		c d			
		00	01	11	10
a b	00				
	01	1	1		
	11	1		1	
	10			1	

f1

		c d			
		00	01	11	10
a b	00	1	1		
	01		1		
	11			1	
	10			1	

f2

$$f1 = bc'd' + a'bc' + acd$$

$$f2 = a'b'c' + a'c'd + acd$$

Multiple Outputs (Target Sharing)

		c d			
		00	01	11	10
a b	00				
	01	1	1		
	11	1		1	
	10			1	

g1

		c d			
		00	01	11	10
a b	00	1	1		
	01		1		
	11			1	
	10			1	

g2

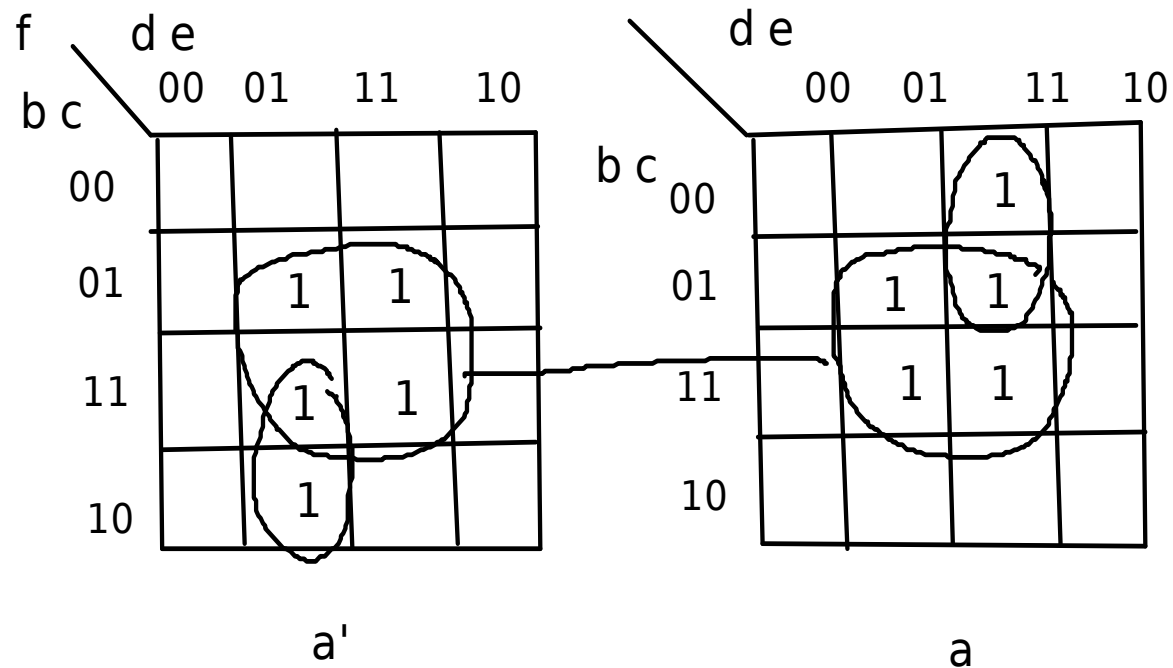
$$g1 = bc'd' + a'bc'd + acd$$

$$g2 = a'b'c' + a'bc'd + acd$$

Realization Cost

- TTL ($f1 f2$)
 - 5 3-input AND gates
 - 2 3-input OR gates
 - 9 1-input NOT gates
- TTL ($g1 g2$)
 - 3 3-input AND gates
 - 1 4-input AND gates
 - 2 3-input OR gates
 - 7 1-input NOT gates
- PLA ($f1 f2$)
 - 5 product terms
- PLA ($g1 g2$)
 - 4 product terms

Example (5 Variables)



$$f = ce + a'bd'e + ab'de$$