

Hopcroft's algorithm

- We want to identify and merge any equivalent states in the DFA: states produce identical behaviour across all inputs
- It takes the set of all states, and repeatedly partitions them into smaller and smaller subsets by identifying ways in which some states in a subset behave differently than others in the subset
- First: split the set of all states into accept/non-accept
- Repeat: pick an input character, check if each state in a partition takes you to a common partition (e.g. on input x , do all states in partition P transition to states in partition Q)

The algorithm

Current = { AcceptStates, NonAcceptStates }

P = { }

Repeat until P and Current are the same:

 P = Current

 Current = P

 For each set, s, in P

 Current = Union of Current and Split(s)

(split tries to break s into two sets of “distinguishable” states)

Split(s)

for each input character, c:

 for each state in s

 determine which partition $T[s][c]$ leads to

 if some states of s lead to one partition

 and other states do not, use that to

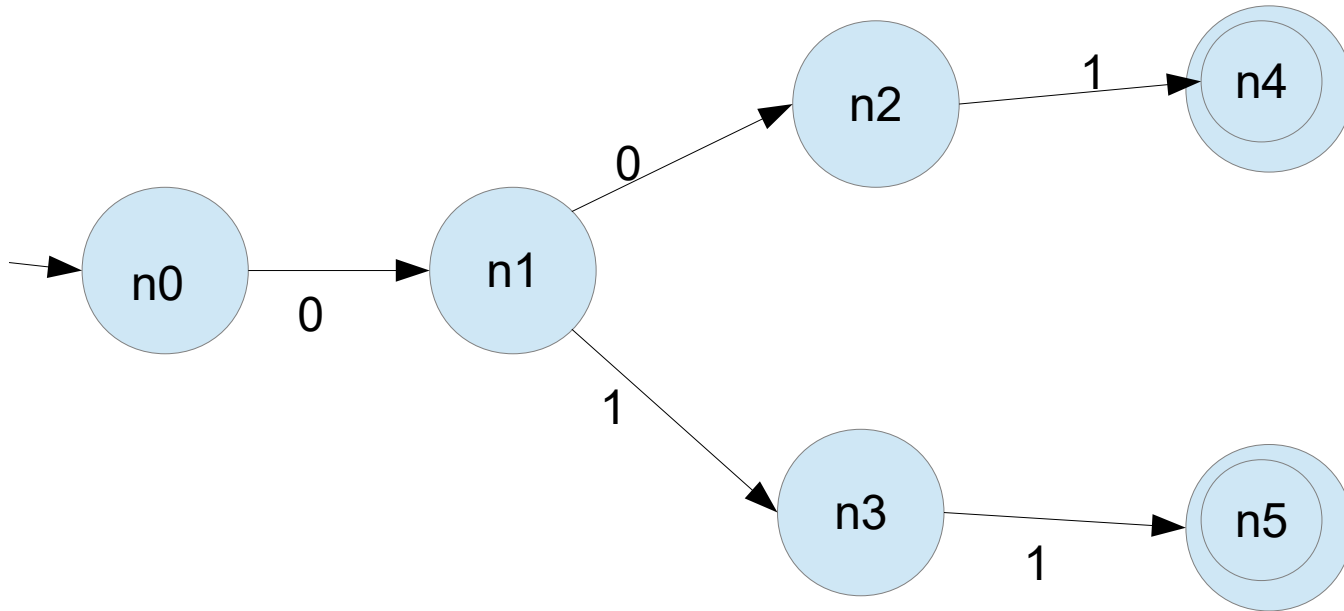
 divide them into two sets, s1 and s2

 and return { s1, s2 }

return S

Example 1

- Will try to minimize the following DFA



Example 1

First, divide into accept/non { {n0,n1,n2,n3}, {n4,n5} }

Make first pass through repeat loop

in first pass of for loop, try Split({n0,n1,n2,n3})

on input 0, n2/n3 go to states in p1, n0/n1 do not

Current = { {n0,n1}, {n2,n3} }

in second pass of for loop, try Split({n4,n5})

no differences found between n4/n5 on either input

Current = { {n0,n1}, {n2,n3}, {n4,n5})

Example 1 continued

Second pass of repeat loop

First for loop pass, try $\text{Split}(\{n_0, n_1\})$

On input 1, $n_0 \rightarrow n_1$, $n_1 \rightarrow n_2$, in different partitions

Current = $\{ \{n_0\}, \{n_1\} \}$

Second for loop pass, try $\text{Split}(\{n_2, n_3\})$

No changes found

Current = $\{ \{n_0\}, \{n_1\}, \{n_2, n_3\} \}$

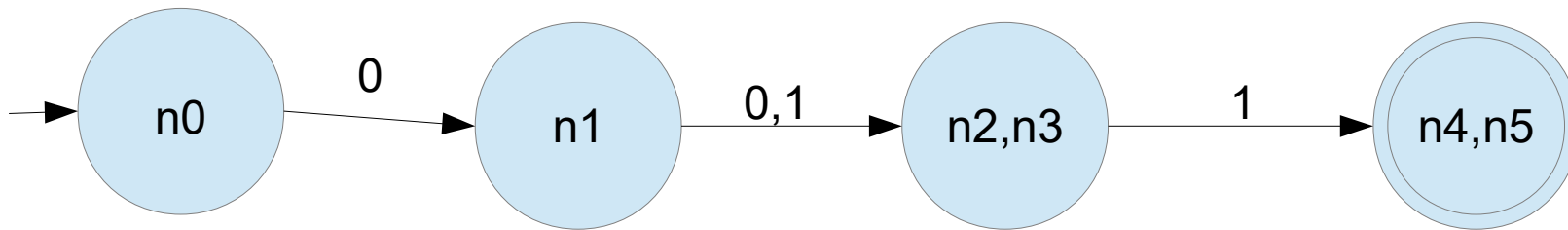
Third for loop pass, try $\text{Split}(\{n_4, n_5\})$

No changes found

Current = $\{ \{n_0\}, \{n_1\}, \{n_2, n_3\}, \{n_4, 5\} \}$

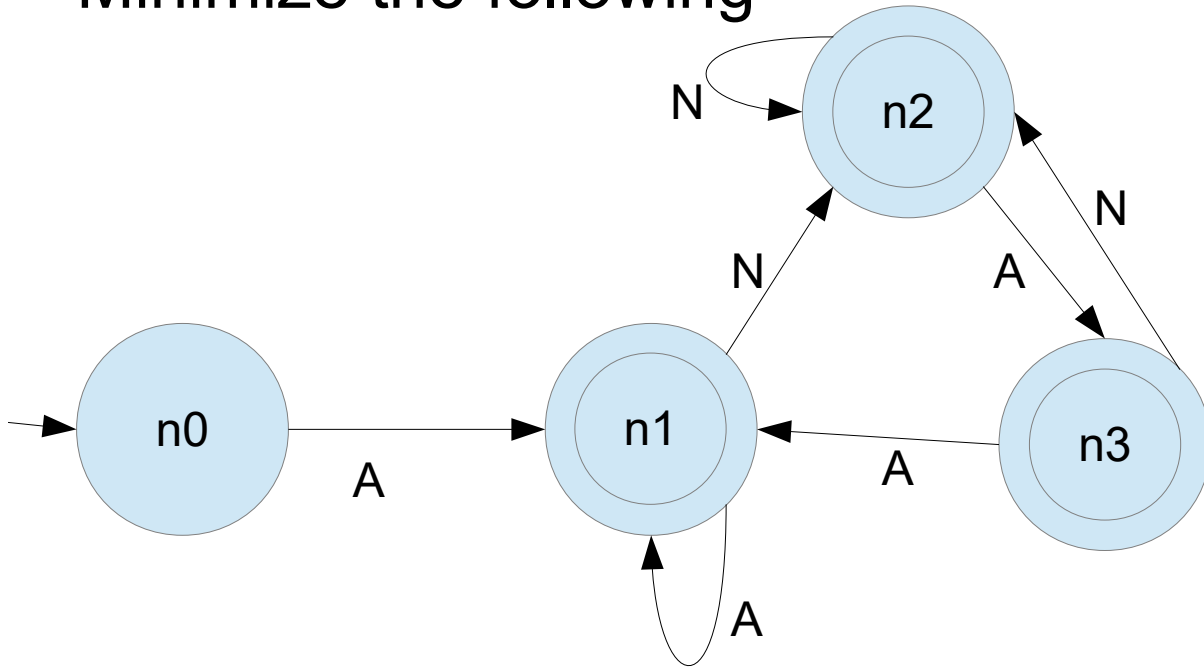
Example 1: final

- Can end now that P and $Current$ are the same, final partitioning realizes n_2, n_3 are equivalent, and that s_4, s_5 are equivalent, giving minimized DFA



Example 2

- Minimize the following



Example 2 continued

Current = { {n0}, {n1,n2,n3} }

Split({n0}) can't possibly split further

Split({n1,n2,n3})

For every state/input combination we get one of n1,n2,n3

Can't subdivide further, current is actually the right partition

